TOPOLOGY II SEMESTRAL EXAM

Time : (3+δ) hours Max. Marks : 60

Answer all questions. You may use results proved in class after correctly quoting them. Any other claim must be accompanied by a proof.

(1) Decide whether the following statements are *true* or *false*. Answers without correct and complete justifications will not be awarded any marks.
 (a) The Mobius band $M = [0,1] \times [0,1]/(0,t) \sim (1, 1-t)$ retracts onto its boundary circle $S = \{[s,t] : 0 \leq s \leq 1, t = 0, 1\}$.
 (b) The inclusion map $j : (D^n, S^{n-1}) \longrightarrow (\mathbb{R}^n, \mathbb{R}^n - 0)$ is a homotopy equivalence.
 (c) If F is a free group of rank 2 and N a normal subgroup of infinite index, then N cannot be finitely generated.
 (d) There exists a CW-structure on $S^1 \times S^2$ for which all the boundary homomorphisms in the cellular chain complex of $S^1 \times S^2$ are zero.
 (e) There exists a continuous map $f : \mathbb{R}P^3 \longrightarrow \mathbb{R}P^3$ of degree 6. [4xe=20!]

(2) Let F denote the free group on two generators a, b and G the subgroup $G = \langle a^2, b^2, (ab)^2, (ba)^2, ab^2a \rangle$. Determine the index of G in F. Is G normal? [7+3]

(3) Define the term CW-complex. Show that the complex projective space $\mathbb{C}P^n$ is a CW-complex. Compute the homology groups of $\mathbb{C}P^n/\mathbb{C}P^m$ for $m \leq n$. [2+6+2]

(4) Compute the degree of the antipodal map $a : S^n \longrightarrow S^n$. [10]

(5) Let B be a k-cell in S^n. Show that B does not separate S^n. [10]